Arthur Hoag, “Aristarchos Revisited.” *Griffith Observer*, 54 (8; August, 1990): 10-18.

Aristarchos Revisited

A. Hoag
Tucson, Arizona

Nicolaus Copernicus gets a lot of credit for putting us on a path around the sun, and he deserves it. Long before the sixteenth century, however, the ancient Greeks had an advocate for a sun-centered solar system. That geo-discentrist was Aristarchos. We know that he lived during the third century B.C. and came from the island of Samos, in the east Aegean Sea. Aristarchos managed to estimate the size of the sun, compared it with the size of the earth, and concluded the sun is much larger. Because the sun is larger, he reasoned, it had to be the center of the Greek universe. The sun is not at the center of the universe, of course, but it is, for all practical purposes, at the center of the solar system. The earth travels around it, just as Aristarchos said it did. There is, however, a problem with this 23-centuries-old breakthrough in scientific knowledge. The method Aristarchos said he employed to measure the size and distance of the sun and moon doesn't really seem to work in practical application. The measurements actually involve very small angles, and the techniques Aristarchos had at his disposal would have introduced significant errors. For that reason, some scholars have claimed that Aristarchos fabricated his data to fit a preconceived conclusion. Dr. Art Hoag, whose distinguished career in astronomy during this century has not centered on the sun's status in the solar system, does have concrete experience with the trials and triumphs of observational technique. Perhaps for that reason he stepped to the front lines in defense of the ancient astronomers and attempted to duplicate the measurements Aristarchos might have made. This is an intriguing exercise in experimental history with results that celebrate economy of thought.

I have been intrigued by the Aristarchos story for a very long time but have only recently begun to learn some of the details. We know something about his notions of the solar system because his treatise of about 280 B.C., *On the Sizes and Distances of the Sun and Moon,* has come down to us "in its integrity," as historians say. We know even more about his ideas because of Archimedes's commentary in *The Sand Reckoner*[1]:

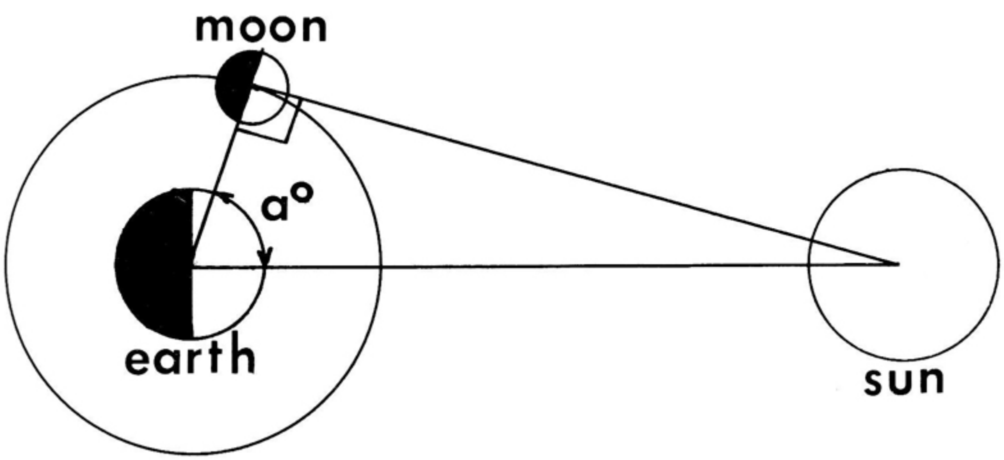
> **You [Galen II, king of Syracuse] are aware that universe is the name given by most astronomers to the sphere whose center is the center of the earth, and whose radius is equal to the distance between the center of the sun and the center of the earth. This is the common account as you have heard from astronomers. But Aristarchos of Samos brought out a book consisting of some hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the one just mentioned. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same center as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface. [In other words, the entire orbit of the earth would appear as a point from the surface of the sphere of fixed stars]**

George Sarton goes on to say[2], after quoting the above:

> **This is stupendous, and would be incredible if we had it from another source, but we have no reason to doubt Archimedes, who was born within Aristarchos's lifetime and might have known him personally. Moreover, why would he invent such a statement? Or, if he invented it, it would be just as stupendous.**

Archimedes was enthusiastic about this larger concept of the universe because he wanted to demonstrate his facility with large numbers by calculating the number of grains of sand one could pack into the universe.

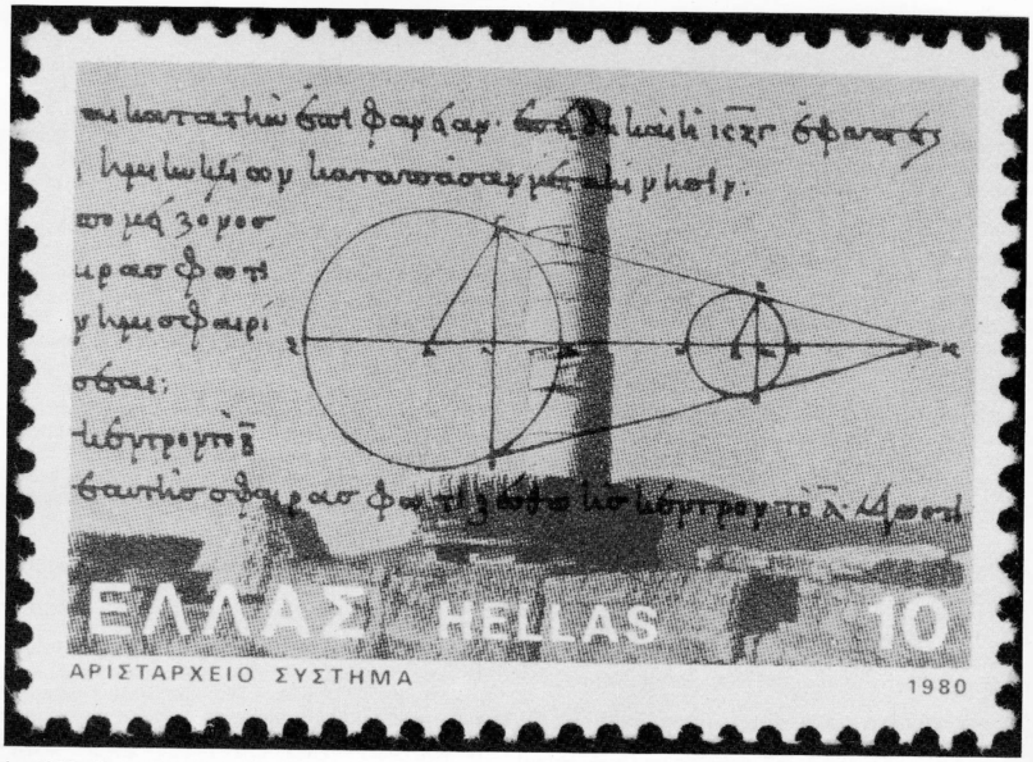
The hypothesis in the Aristarchos treatise that is the key to the relative sizes and distances of the sun and moon is: "That, when the moon appears to us halved, its distance (elongation) from the sun is less than a quadrant by one-thirtieth of a quadrant [87°]." It is quite logical to suppose that this angle should be less than 90° otherwise the sun would have to be at infinity. But why 87°? Could he have made an appropriate measurement?



Aristarchos knew that the sun, moon, and earth occupy the corners of a right triangle "when the moon appears to halved" and that the moon is located at the vertex of the 90-degree angle. It has to be that way, or we would not see the moon in dichotomy (exactly halved) as it reflects light falling on it from the sun. Aristarchos also realized that angle a *must be less than 90°. If not, the line connecting the sun and moon would be parallel with the line connecting the sun and earth, and that would mean the sun is an infinite distance away. The sun isn't infinitely far, however, and Aristarchos said angle a is 87°. It isn't. It is 89° 51′ — much closer to 90° and so slightly different from 90° that Aristarchos could not have measured it with precision. An 87-degree angle told him, however, that the sun is large and farther than the moon. Would his instruments and methods have given him that 87-degree angle, or did he just make it up?* (drawing by Dr. A. Hoag)

I had once imagined that Aristarchos had observed the moon to be "halved" (this is called dichotomy) when it was three degrees west of the meridian at sunset. This wouldn't work very well because of refraction which wasn't understood at that time. The observed positions of the sun and moon are slightly displaced from their true positions by the atmosphere's ability to bend, or refract, light. Further, the sun and moon would both have to be on the celestial equator (which doesn't happen at quadrature) or corrections have to be made for differing declinations. Aristarchos could have used setting or transit times, correcting for apparent motions during the intervals between sunset and moonset or transit, but, again, what we need to solve the problem correctly is the moon's elongation in celestial longitude — its angular distance from the sun along the ecliptic — not the moon's hour angle. Some authors have suggested that Aristarchos used the inequality of times between first and third quarter and third and first quarter, but if he somehow managed to get a useful result, it would have been an accident. That is because the moon's apparent motion is not uniform. He could have used ephemeris times of quadrature, a Babylonian heritage, compared to estimated times of dichotomy. Quadrature occurs when the moon is 90° from, or at right angle to, the sun. He could have directly measured the angle between the sun and the moon with a gnomon at estimated half-phase times. Finally, he could have picked a reasonable figure out of thin air to solve a hypothetical problem with the impressively complex Euclidian means at his disposal. See what Otto Neugebauer, a leading authority on ancient mathematics, had to say about this[3]:

> **We now come to the astronomical analysis of Aristarchus' treatise. It is obvious that its fundamental idea, the use of the elongation at the moment of dichotomy, is totally impractical. The elongation of the moon changes one degree in about two hours; thus one should be able to establish the moment of dichotomy within at least one hour.**
>
> **In fact, one would be lucky to determine the night in which dichotomy falls; hence**



In 1980, Greece honored Aristarchos by issuing a pair of commemorative stamps. One of them shows the geometric configuration he used to ascertain the relative size of the sun. This diagram is superimposed upon a view of the only standing column of the Temple of Hera on the island of Samos. The second, on the next page, is a modern portrait of the planets' orbits centered on the sun. (collection E.C. Krupp)

> 87° **is a purely fictitious number. *The actual value is about 89° 51′ and must therefore elude direct determination by ancient observers.***

Being an ancient observer myself, I took umbrage at this last remark and felt challenged to have a try at measurements Aristarchos might have made.

Of the methods outlined previously, I guessed that direct measures of the angle between the sun and the moon, around times of quarter moon, and at times when the sun and moon were at equal altitudes in the daylight sky, would be best. I used an eight-foot vertical pole as a gnomon. I must confess that the auxiliary apparatus I used was non-Aristarchan; a WWV receiver, a steel tape, my wife's compact mirror for sighting the moon over the top of the pole, and a calculator for trigonometric functions. Repetitive observations indicated that the internal accuracy of the measures of the sun-earth-moon angle was good to a tenth of a degree.

Concurrent altitude measures of the sun and moon, corrected for refraction and finite gnomon diameter, showed no appreciable systematic error.

I actually made sequences of measures of the sun-earth-moon angle on several (average of four) days around lunar quadrature at two first quarter and three last quarter phases. At the time of each measurement I also estimated the fractional illumination of the moon in two ways; one by pure guesswork — imploring Urania, the Muse of astronomy, to put a number in my head, and the other by making repetitive drawings of the moon that were later planimetered. Surprisingly, divination seemed to work a bit better than measurement of sketches.

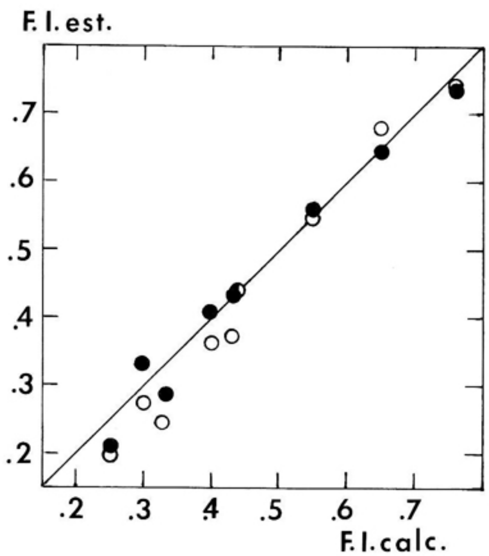
Because rates of change in elongation and in fractional illumination of the moon are quite uniform near the quarter phases, I could

interpolate linear regressions to get the desired sun-earth-moon angles at times of dichotomy. The data and result for the example given in the accompanying diagram are shown in Table 1. The mean result for all five quarter-phase series was 89°.9 ± 0°.6 S.D. The predicted value of the dichotomy angle for the mean distances of the sun and moon is about 89°.852, hence the sun is about 390 times the lunar distance. My result suggests that the sun is probably not less than tangent 89°.3 = 82 times as distant as the moon and may be, more likely, tangent 89°.09 = >500 times more distant. Aristarchos's use of a dichotomy angle of "less than a quadrant by one-thirtieth of a quadrant", and the constraint of his techniques in geometry, led to a result of greater than 18 and less than 20 for the sun-to-moon distance ratio, wrong by a factor of about 20. He could have done better with observing techniques then available but he didn't have to do better. He realized from his result that the sun had to be the dominant body and that the earth must revolve around it. That, as reported by Archimedes, together with no apparent parallax for the stars, indicated that the universe was vast as compared to previous concepts.

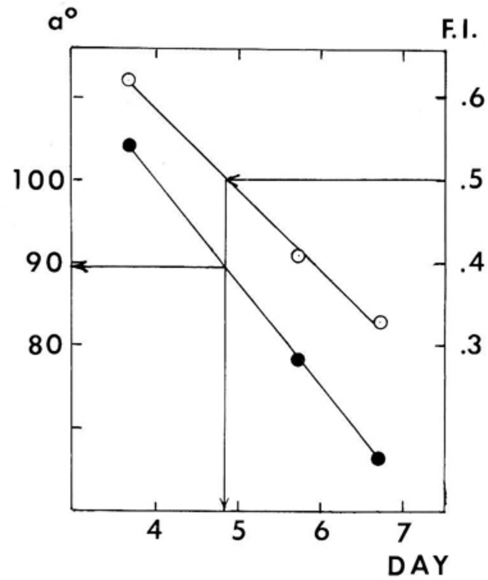
I did try one four-day series of transit measures as a means of determining the dichotomy angle at one first quarter phase but missed the calculated value by about one degree (two hours from dichotomy).

The method of lunar dichotomy was used for a surprisingly long time, even by Kepler — perhaps as a gesture to those who didn't understand the implications of his ideas about planetary orbits! A history of application of the method is summarized in Table 2. Before Aristarchos, Eudoxus (a pupil of Plato) and Phidias (the father of Archimedes) had proposed that the sun was ten times more distant than the moon and as much larger. They also knew from Pythagoras and from Aristotle the form and approximate circumference of the earth and could deduce that the sun was larger than the earth. There is no evidence of analytical bases for their estimates and they did not propose that the earth revolved around the sun. Aristarchos, however, also "stood on the shoulders of giants."

In the first century B.C., Posidonius obtained a remarkably good estimate. The "magic" of the Posidonius result consisted in his apparently



This diagram shows how well a careful observer can determine when the moon is exactly half-illuminated. The black dots represent visual estimates of the shining fraction of the moon's disk. Dr. Hoag also made careful drawings of the moon in various configurations and measured them with a planimeter — an instrument unavailable to Aristarchos — to determine what fraction of the disk was lit. These observations are represented by the open circles. The graph compares these estimates (vertical axis) with accurate calculations, based on our modern knowledge of the moon's motion, of the fractional illumination (horizontal axis). The diagram shows that thoughtful eyeball estimates aren't that bad — good enough, in fact, to get an answer even closer to the truth than Aristarchos did. (drawing by Dr. A. Hoag)



This diagram allows us to see how well a series of observations close to the last quarter moon will pin down the date and time of an exactly half-full moon. This time the solid dots stand for angle a (sun-earth-moon), and the open circles are estimates of the fraction of the moon's disk illuminated by sunlight. The fraction of sunlight is indicated by the vertical axis, and so is angle a. Time (in days) from an arbitrary reference date is given on the horizontal axis. These data tell us that in this case, the half-lit moon corresponds to day 4.8629 and gives an angle a = 89.4°, slightly less than a half-degree error and a respectably accurate observation. (drawing by Dr. A. Hoag)

arbitrary selection of two round numbers for the distances of the sun and moon: 500 x 10^6 and 2 x 10^6 stadia.

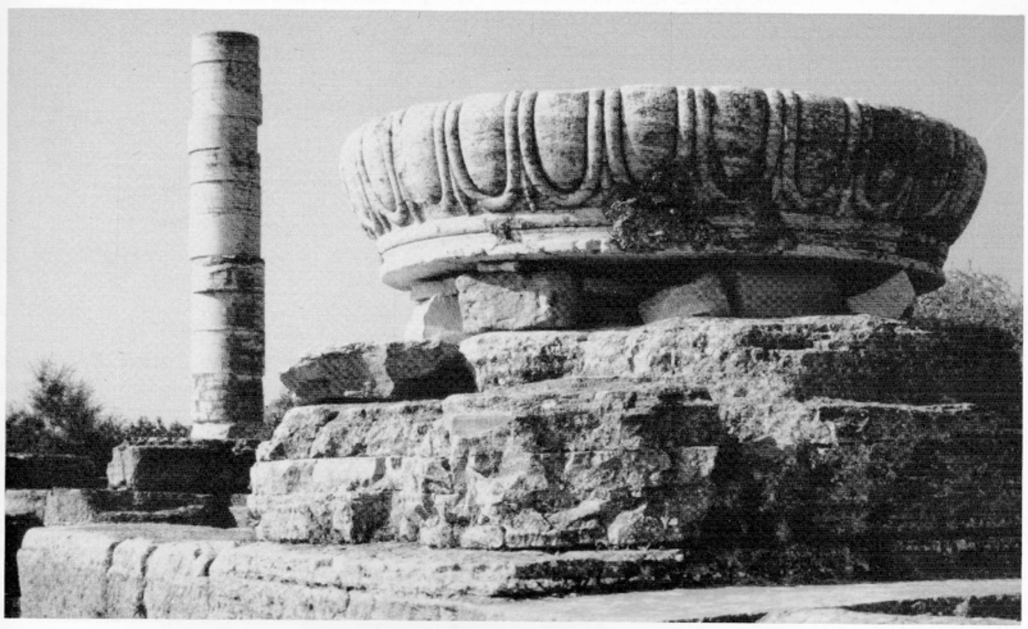
Better estimates of the sun-earth-moon angle at dichotomy were made after the advent of the telescope, beginning with Harriot. No great accuracy could ever be achieved however, because it was difficult to estimate the moment of dichotomy. The terminator is made too irregular by lunar topography to permit precise measurement.

The observations reported by Riccioli were probably more a result of correspondence with Wendelin and van Langren than a result of unbelievably precise measures. Rocca's estimate shows the primary weakness in the lunar dichotomy method. Any imprecision in estimating an angle near 90° will result in a large change in the moon-sun distance ratio.

The improved values for the relative distances of the moon and sun after 1640 may be more a result of a growing awareness of the true value of the solar parallax derived by other methods.

To recapitulate: First, Aristarchos seems to have been a theoretician, not an observer. By solving a hypothetical realistic problem, he came to the conclusion that the earth revolved around the sun and that the universe could be scaled by the size of that orbit. Second, the error in my observations of the dichotomy angle affirms Alan Batten's prediction that "an uncertainty of about an hour (in the time of dichotomy) is inevitable with naked eye observation." Finally, third, estimate or measurement of dichotomy times is a poor way of getting the distance to the sun, as the record

Samos — the home of Aristarchos — is in the eastern Aegean, just a mile or so off Turkey's Ionian shore. This view of Samos includes the town of Pythagorion, on the southeast side of the island. Pythagorion is named, of course, for another famous son of Samos — Pythagoras, the mathematician and philosopher whose name is bonded to the theorem that details the relationship between the sides of a right triangle. Pythagoras was born on Samos in the sixth century B.C. (photograph E.C. Krupp)



The Heraion, or Temple of Hera, on Samos was built about three centuries before the time of Aristarchos. Only one of the temple's original marble columns is still standing. (photograph E.C. Krupp)

Table 1

Last Quarter - August, 1988

U.T. Day	sun-moon angle	fractional illumination
(d)	(a)	(F.I.)
3.6583	104°.3	
3.6625	104°.2	
3.6667		0.62
3.6708	104°.1	
3.6778	104°.2	
<3.6674>	<104°.2>	
5.7104	78°.8	0.42
5.7118	78°.8	
5.7167	78°.8	0.40
5.7229	78°.6	
5.7330	78°.6	
<5.7190>	<78°.7>	
6.6958	66°.6	0.32
6.7007	66°.5	
6.7146	66°.4	0.34
6.7319	66°.4	
6.7375	66°.2	
<6.7161>	<66°.4>	

Linear Regression

$$a = -12.4031\ d + 149.673$$

$$F.I. = -0.09548\ d + 0.9643$$

at $F.I. = 0.5$, $d = 4.8629$

at $d = 4.8629$, $a = 89°.4$ (the dichotomy angle)

of history shows, but the method was good enough to lead Aristarchos to "stupendous" conclusions.

References

1 From Heath's translation in his *Works of Archimedes,* Cambridge, 1897, p. 221. Note: Archimedes wrote *The Sand Reckoner* before 216 B.C., the year of king Galen's death. Archimedes died in 212 B.C. at the age of 75.

2 From *A History of Science: Hellenistic Science and Culture in the Last Three Centuries B.C.,* George Sarton, Harvard, 1959, p. 57.

3 From *A History of Ancient Mathematical Astronomy,* Otto Neugebauer, Springer Verlag, 1975, p. 642. (The emphasis is mine.)

4 Alan Batten, *Roy. Astr. Soc. Canada,* Vol. 75, p. 29, 1981.

Further Reading

Aristarchus of Samos: The Ancient Copernicus, by Sir Thomas Heath (Oxford University Press, 1913) has been reprinted by Dover (1981).

There are a great many versions of the Aristarchos story. Among them I enjoyed the Sarton and Batten accounts cited above and especially John Irwin's prize winning "The Case of the Celestial Shadows" in the October, 1977, *Griffith Observer.* See also Owen Gingerich's articles in the November, 1980, and November, 1981, *Sky and Telescope.*

I made extensive use of Albert van Helden's

Table 2

The dichotomy angle and sun/moon distance ratio.

Eudoxus	380 B.C.	84°	9
Phidias	290	85°	11
Aristarchos	280	87°	18, 20
Archimedes	240	88°.1	30
Hipparchus	128	88°.5	40
Posidonius	90	89°.8	290 magic!
Ptolemy	150 A.D.	87°.2	20
Copernicus	1540	87°.7	25
Harriot	1611	89°.1	64
Kepler	1615	88°.1	30
Wendelin	1644	89°.75	230
	1647	89°.5	120
Van Langren	1640s	89°.0	60
Riccioli	1646	89° 28′ 26″	109 absurd !
		89° 29′ 45″114	
Rocca	1640s	90°	infinity!
Streete	1661	89°.75	230
<actual>	now	89°.852	387

excellent *Measuring the Universe* (University of Chicago Press, 1985) in tracing the history of the application of the Aristarchos lunar dichotomy method.